

## SPM BULLETIN

## ISSUE NUMBER 38: Nov 2015 CE

## 1. EDITOR'S NOTE

The *International Conference on Topology* took place in Messina, September 7–11, 2015. A substantial portion of the lectures dealt with selection principles. The program and book of abstracts are available on the conference webpage, <http://mat521.unime.it/ictm2015>.

A special issue of the journal *Topology and its Applications* will be dedicated to the conference's themes and, in particular, to selection principles. The guest editors for this issue are Maddalena Bonanzinga and Boaz Tsaban. Papers meeting the journal's high standards may be submitted, by the end of January (tentative), to the special issue's email address [ictmessina2015@gmail.com](mailto:ictmessina2015@gmail.com). The papers will be fully refereed according to the journal's standards. Attendance in the conference is not a prerequisite for submission; the sole criteria are quality and relevance (in this order).

With best regards,

Boaz Tsaban, [tsaban@math.biu.ac.il](mailto:tsaban@math.biu.ac.il)  
<http://www.cs.biu.ac.il/~tsaban>

## 2. LONG ANNOUNCEMENTS

**2.1. Selective versions of chain condition-type properties.** We study selective and game-theoretic versions of properties like the ccc, weak Lindelöfness and separability, giving various characterizations of them and exploring connections between these properties and some classical cardinal invariants of the continuum.

<http://arxiv.org/abs/1502.00177>

Leandro Aurichi, Santi Spadaro, Lyubomyr Zdomskyy

**2.2. On topological properties of the weak topology of a Banach space.** Being motivated by the famous Kaplansky theorem we study various sequential properties of a Banach space  $E$  and its closed unit ball  $B$ , both endowed with the weak topology of  $E$ . We show that  $B$  has the Pytkeev property if and only if  $E$  in the norm topology contains no isomorphic copy of  $\ell_1$ , while  $E$  has the Pytkeev property if and only if it is finite-dimensional. We extend Schlüchtermann and Wheeler's result by showing that  $B$  is a (separable) metrizable space if and only if it has countable  $cs^*$ -character and is a  $k$ -space. As a corollary we obtain that  $B$  is Polish if and only if it has countable  $cs^*$ -character and is Čech-complete, that supplements a result of Edgar and Wheeler.

<http://arxiv.org/abs/1502.00178>

Saak Gabrielyan, Jerzy Kakol, Lyubomyr Zdomskyy

**2.3. On metrizable  $X$  with  $C_p(X)$  not homeomorphic to  $C_p(X) \times C_p(X)$ .** We give two examples of infinite metrizable spaces  $X$  such that the space  $C_p(X)$ , of continuous real-valued function on  $X$  endowed with the pointwise topology, is not homeomorphic to its own square  $C_p(X) \times C_p(X)$ . The first of them is a one-dimensional continuum; the second one is a zero-dimensional subspace of the real line. Our result answers a long-standing open question in the theory of function spaces posed by A.V. Arhangel'skii.

<http://arxiv.org/abs/1503.04229>

*Mikołaj Krupski and Witold Marciszewski*

**2.4. On complete metrizable space of the Hausdorff metric topology.** There exists a completely metrizable bounded metrizable space  $X$  with compatible metrics  $d, d'$  so that the hyperspace  $CL(X)$  of nonempty closed subsets of  $X$  endowed with the Hausdorff metric  $H_d, H_{d'}$ , resp. is  $\alpha$ -favorable,  $\beta$ -favorable, resp. in the strong Choquet game. In particular, there exists a completely metrizable bounded metric space  $(X, d)$  such that  $(CL(X), H_d)$  is not completely metrizable.

<http://arxiv.org/abs/1503.04383>

*Laszlo Zsilinszky*

**2.5.  $G_\delta$  semifilters and  $\omega^*$ .** The ultrafilters on the partial order  $([\omega]^\omega, \subseteq^*)$  are the free ultrafilters on  $\omega$ , which constitute the space  $\omega^*$ , the Stone-Čech remainder of  $\omega$ . If  $U$  is an upper set of this partial order (i.e., a *semifilter*), then the ultrafilters on  $U$  correspond to closed subsets of  $\omega^*$  via Stone duality. If, in addition,  $U$  is sufficiently "simple" (more precisely,  $G_\delta$  as a subset of  $2^\omega$ ), we show that  $U$  is similar to  $[\omega]^\omega$  in several ways. First,  $\mathfrak{p}_U = \mathfrak{t}_U = \mathfrak{p}$  (this extends a result of Malliaris and Shelah). Second, if  $\mathfrak{d} = \mathfrak{c}$  then there are ultrafilters on  $U$  that are also  $P$ -filters (this extends a result of Ketonen). Third, there are ultrafilters on  $U$  that are weak  $P$ -filters (this extends a result of Kunen). By choosing appropriate  $U$ , these similarity theorems find applications in dynamics, algebra, and combinatorics. Most notably, we will answer two open questions of Hindman and Strauss by proving that there is an idempotent of  $\omega^*$  that is both minimal and maximal.

<http://arxiv.org/abs/1503.06092>

*Will Brian and Jonathan Verner*

**2.6. Selective strong screenability and a game.** Selective versions of screenability and of strong screenability coincide in a large class of spaces. We show that the corresponding games are not equivalent in even such standard metric spaces as the closed unit interval. We identify sufficient conditions for ONE to have a winning strategy, and necessary conditions for TWO to have a winning strategy in the selective strong screenability game.

<http://arxiv.org/abs/1503.08467>

*Liljana Babinkostova and Marion Scheepers*

**2.7. Cofinality spectrum theorems in model theory, set theory, and general topology.** We connect and solve two long-standing open problems in quite different areas: the model-theoretic question of whether  $\text{SOP}_2$  is maximal in Keisler's order, and the question from general topology/set theory of whether  $\mathfrak{p} = \mathfrak{t}$ , the oldest problem on cardinal invariants of the continuum. We do so by showing these problems can be translated into instances of a more fundamental problem which we state and solve completely, using model-theoretic methods.

[www.ams.org/journal-getitem?pii=S0894-0347-2015-00830-X](http://www.ams.org/journal-getitem?pii=S0894-0347-2015-00830-X)

*Journal of the American Mathematical Society*

M. Malliaris; S. Shelah

**2.8. Menger remainders of topological groups.** In this paper we discuss what kind of constraints combinatorial covering properties of Menger, Scheepers, and Hurewicz impose on remainders of topological groups. For instance, we show that such a remainder is Hurewicz if and only if it is  $\sigma$ -compact. Also, the existence of a Scheepers non- $\sigma$ -compact remainder of a topological group follows from CH and yields a  $P$ -point, and hence is independent of ZFC. We also make an attempt to prove a dichotomy for the Menger property of remainders of topological groups in the style of Arhangel'skii.

<http://arxiv.org/abs/1504.01626>

*Angelo Bella, Seçil Tokgöz, and Lyubomyr Zdomskyy*

**2.9. The Whyburn property and the cardinality of topological spaces.** The weak Whyburn property is a generalization of the classical sequential property that was studied by many authors. A space  $X$  is weakly Whyburn if for every non-closed set  $A \subset X$  there is a subset  $B \subset A$  such that  $\overline{B} \setminus A$  is a singleton. We prove that every countably compact Urysohn space of cardinality smaller than the continuum is weakly Whyburn and show that, consistently, the Urysohn assumption is essential. We simultaneously solve a question of Pelant, Tkachenko, Tkachuk and Wilson and one of Bella, Costantini and ourselves by constructing a Lindelöf  $P$ -space of cardinality  $\omega_2$  that is not weakly Whyburn. We give conditions for a weak Whyburn space to be pseudoradial and construct a countably compact weakly Whyburn non-pseudoradial regular space, which solves a question asked by Bella in private communication.

<http://arxiv.org/abs/1505.06238>

*Santi Spadaro*

**2.10. A note on local properties in products.** We give conditions under which a product of topological spaces satisfies some local property. The conditions are necessary and sufficient when the corresponding global property is preserved under finite products. Further examples include local sequential compactness, local Lindelöfness, the local Menger property.

<http://arxiv.org/abs/1506.00224>

*Paolo Lipparini*

**2.11. Point-open games and productivity of dense-separable property.**

<http://arxiv.org/abs/1506.06080>

*Jarno Talponen* In this note we study the point-open topological games to analyze the least upper bound for density of dense subsets of a topological space. This way we may also analyze the behavior of such cardinal invariants in taking products of spaces. Various related cardinal equalities and inequalities are given. As an application we take a look at Banach spaces with the property (CSP) which can be formulated by stating that each weak-star dense linear subspace of the dual is weak-star separable.

**2.12. Infinite games and chain conditions.** We apply the theory of infinite two-person games to two well-known problems in topology: Suslin's Problem and Arhangel'skii's problem on  $G_\delta$  covers of compact spaces. More specifically, we prove results of which the following two are special cases: 1) every linearly ordered topological space satisfying the game-theoretic version of the countable chain condition is separable and 2) in every compact space satisfying the game-theoretic version of the weak Lindelöf property, every cover by  $G_\delta$  sets has a continuum-sized subcollection whose union is  $G_\delta$ -dense.

<http://arxiv.org/abs/1507.02134>

*Santi Spadaro*

**2.13. Nonmeasurable sets and unions with respect to selected ideals especially ideals defined by trees.**

<http://arxiv.org/abs/1507.02496>

*Robert Ralowski and Szymon Zeberski* In this paper we consider nonmeasurability with respect to  $\sigma$ -ideals defined by trees. First classical example of such ideal is Marczewski ideal  $s_0$ . We will consider also ideal  $l_0$  defined by Laver trees and  $m_0$  defined by Miller trees. With the mentioned ideals one can consider  $s$ ,  $l$  and  $m$ -measurability.

We have shown that there exists a subset  $A$  of the Baire space which is  $s$ ,  $l$  and  $m$  nonmeasurable at the same time. Moreover,  $A$  forms m.a.d. family which is also dominating. We show some examples of subsets of the Baire space which are measurable in one sense and nonmeasurable in the other meaning.

We also examine terms nonmeasurable and completely nonmeasurable (with respect to several ideals with Borel base). There are several papers about finding (completely) nonmeasurable sets which are the union of some family of small sets. In this paper we want to focus on the following problem: Let  $P$  be a family of small sets. Is it possible that for all  $A$  which is a subset of  $P$ , union of  $A$  is nonmeasurable implies that union of  $A$  is completely nonmeasurable?

We will consider situations when  $P$  is a partition of  $R$ ,  $P$  is point-finite family and  $P$  is point-countable family. We give an equivalent statement to CH using terms nonmeasurable and completely nonmeasurable.

**2.14. Some observations on the Baireness of  $C_k(X)$  for a locally compact space  $X$ .** We prove some consistency results concerning the Moving Off Property for locally compact spaces and thus the question of whether their function spaces are Baire.

<http://arxiv.org/abs/1507.06717>

*Franklin D. Tall*

**2.15. Reconstructing Compact Metrizable Spaces.** The deck,  $\mathcal{D}(X)$ , of a topological space  $X$  is the set  $\mathcal{D}(X) = \{[X \setminus \{x\}] : x \in X\}$ , where  $[Y]$  denotes the homeomorphism class of  $Y$ . A space  $X$  is (topologically) reconstructible if whenever  $\mathcal{D}(Z) = \mathcal{D}(X)$  then  $Z$  is homeomorphic to  $X$ . It is known that every (metrizable) continuum is reconstructible, whereas the Cantor set is non-reconstructible.

The main result of this paper characterises the non-reconstructible compact metrizable spaces as precisely those where for each point  $x$  there is a sequence  $\langle B_n^x : n \in \mathbb{N} \rangle$  of pairwise disjoint clopen subsets converging to  $x$  such that  $B_n^x$  and  $B_n^y$  are homeomorphic for each  $n$ , and all  $x$  and  $y$ .

In a non-reconstructible compact metrizable space the set of 1-point components forms a dense  $G_\delta$ . For  $h$ -homogeneous spaces, this condition is sufficient for non-reconstruction. A wide variety of spaces with a dense  $G_\delta$  set of 1-point components are presented, some reconstructible and others not reconstructible.

<http://arxiv.org/abs/1510.02654>

*Paul Gartside, Max F. Pitz, Rolf Suabedissen*

**2.16. On topological groups admitting a base at identity indexed with  $\omega^\omega$ .** A topological group  $G$  is said to have a  $\mathfrak{G}$ -base if the neighbourhood system at identity admits a monotone cofinal map from the directed set  $\omega^\omega$ . In particular, every metrizable group is such, but the class of groups with a  $\mathfrak{G}$ -base is significantly wider. The aim of this article is to better understand the boundaries of this class, by presenting new examples and counter-examples. Ultraproducts and non-archimedean ordered fields lead to natural families of non-metrizable groups with a  $\mathfrak{G}$ -base which nevertheless have the Baire property. More examples come from such constructions as the free topological group and the free Abelian topological group of a Tychonoff (more generally uniform) space, as well as the free product of topological groups. Our results answer some questions previously stated in the literature.

<http://arxiv.org/abs/1511.07062>

*Arkady G. Leiderman, Vladimir G. Pestov, and Artur H. Tomita*

### 3. SHORT ANNOUNCEMENTS

**3.1. Strong colorings yield  $\kappa$ -bounded spaces with discretely untouchable points.**

<http://www.ams.org/journal-getitem?pii=S0002-9939-2014-12394-X>

*Istvan Juhasz; Saharon Shelah*

3.2.  $P$ -Paracompact and  $P$ -Metrizable Spaces.<http://arxiv.org/abs/1501.01949>*Ziqin Feng, Paul Gartside, Jeremiah Morgan*

## 3.3. Density character of subgroups of topological groups.

<http://arxiv.org/abs/1501.02877>*Arkady Leiderman, Sidney A. Morris, Mikhail G. Tkachenko*

## 3.4. Nonseparable growth of the integers supporting a measure.

<http://arxiv.org/abs/1501.06972>*Piotr Drygier and Grzegorz Plebanek*3.5. Forcing consequences of  $PFA$  together with the continuum large.<http://www.ams.org/journal-getitem?pii=S0002-9947-2015-06205-9>*David Aspero; Miguel Angel Mota*3.6. On the submetrizability number and  $i$ -weight of quasi-uniform spaces and paratopological groups.<http://arxiv.org/abs/1503.04278>*Taras Banakh and Alex Ravsky*

## 3.7. Verbal covering properties of topological spaces.

<http://arxiv.org/abs/1503.04480>*Taras Banakh and Alex Ravsky*

## 3.8. On the collection of Baire class one functions on the irrationals.

[www.ams.org/journal-getitem?pii=S0002-9939-2015-12583-X](http://www.ams.org/journal-getitem?pii=S0002-9939-2015-12583-X)*Roman Pol*

## 3.9. Measuring sets with translation invariant Borel measures.

<http://arxiv.org/abs/1504.02765>*András Máthé*3.10. Cardinalities of weakly Lindelöf spaces with regular  $G_\kappa$ -diagonals.<http://arxiv.org/abs/1504.01785>*Ivan S. Gotchev*

## 3.11. Generalizations of two cardinal inequalities of Hajnal and Juhász.

<http://arxiv.org/abs/1504.01790>*Ivan S. Gotchev*3.12. Topological properties of function spaces  $C_k(X, 2)$  over zero-dimensional metric spaces  $X$ .<http://arxiv.org/abs/1504.04198>*S. Gabrielyan*

## 3.13. The Ascoli property for function spaces and the weak topology of Banach and Fréchet spaces.

<http://arxiv.org/abs/1504.04202>*S. Gabrielyan, J. Kakol, G. Plebanek*

## 3.14. Ordering A Square.

<http://arxiv.org/abs/1505.02319>*Raushan Z. Buzyakova*

## 3.15. Increasing chains and discrete reflection of cardinality.

<http://arxiv.org/abs/1505.06251>

*Santi Spadaro*

## 3.16. Pinning Down versus Density.

<http://arxiv.org/abs/1506.00206>

*István Juhász, Lajos Soukup, Zoltán Szentmiklóssy*

3.17. Regular  $G_\delta$ -diagonals and some upper bounds for cardinality of topological spaces.

<http://arxiv.org/abs/1506.04665>

*Ivan S. Gotchev, Mikhail G. Tkachenko, and Vladimir V. Tkachuk*

## 3.18. Cardinal invariants distinguishing permutation groups.

<http://arxiv.org/abs/1506.08969>

*Taras Banakh and Heike Mildenberger*

3.19. Cardinality bounds involving the skew- $\lambda$  Lindelöf degree and its variants.

<http://arxiv.org/abs/1507.06684>

*Nathan Carlson, Jack Porter*

3.20. Compact spaces with a  $\mathbb{P}$ -diagonal.

<http://arxiv.org/abs/1508.01541>

*Alan Dow and Klaas Pieter Hart*

## 3.21. Far points and discretely generated spaces.

<http://arxiv.org/abs/1509.01601>

*Alan Dow, Rodrigo Hernández-Gutiérrez*

## 3.22. The weight and Lindelöf property in spaces and topological groups.

<http://arxiv.org/abs/1509.02874>

*Mikhail G. Tkachenko*

## 3.23. On the problem of compact totally disconnected reflection of nonmetrizability.

<http://arxiv.org/abs/1509.05282>

*Piotr Koszmider*

## 3.24. On the sequential closure of the set of continuous functions in the space of separately continuous functions.

<http://arxiv.org/abs/1509.05542>

*Taras Banakh*

3.25. Classifying invariant  $\sigma$ -ideals with analytic base on good Cantor measure spaces.

[www.ams.org/journal-getitem?pii=S0002-9939-2015-12709-8](http://www.ams.org/journal-getitem?pii=S0002-9939-2015-12709-8)

*Taras Banakh; Robert Ralowski; Szymon Zeberski*

## 4. PROBLEM OF THE ISSUE

The undefined terminology is provided at the end of this section. The Isbell–Mrówka  $\Psi$ -spaces are classic examples in the realm of topological covering properties. For obvious reasons (they are not even Lindelöf), Menger’s property is not the right notion to consider in the realm of  $\Psi$ -spaces, and the correct notion, as observed by Bonanzinga and Matveev [1], is the *star-Menger* property. Initial effort to study the question which  $\Psi$ -spaces are star-Menger was put forth in the papers [1, 2]. Despite these, *it is still unknown whether such spaces consistently exist!* More precisely, the only examples of star-Menger spaces known thus far is those of cardinality smaller than  $\mathfrak{d}$ . Such spaces are star-Menger for an obvious counting reason, and are thus trivial examples.

**Problem 4.1** (Bonanzinga–Matveev [1]). *Is there, consistently, a star-Menger  $\Psi$ -space of cardinality  $\geq \mathfrak{d}$ ?*

The problem can be formulated in a combinatorial manner, with no reference to topology. In particular, the definitions below are not necessary in order to consider this problem. This is due to the following result.

**Theorem 4.2** ([2]). *Let  $\mathcal{A} \subseteq P(\mathbb{N})$  be an almost disjoint family. The following assertions are equivalent:*

- (1) *The Isbell–Mrówka space  $\Psi(\mathcal{A})$  is star-Menger.*
- (2) *For each function  $A \mapsto f_A$  from  $\mathcal{A}$  to  $\mathbb{N}^{\mathbb{N}}$ , there are finite sets  $\mathcal{F}_1, \mathcal{F}_2, \dots \subseteq \mathcal{A}$  such that, for each  $A \in \mathcal{A}$ , there is  $n$  with  $(A \setminus f_A(n)) \cap \bigcup_{B \in \mathcal{F}_n} (B \setminus f_B(n)) \neq \emptyset$ .*

It is only known that there are many cardinal numbers with, provably, no star-Menger  $\Psi$ -spaces of that cardinality [1, 2]. In particular, the inequality  $\mathfrak{c} > \aleph_\omega$  must hold in every model witnessing a positive solution of Problem 4.1.

**Basic definitions.** A family  $\mathcal{A} \subseteq P(\mathbb{N})$  is *almost disjoint* if every element of  $\mathcal{A}$  is infinite, and the sets  $A \cap B$  are finite for all distinct elements  $A, B \in \mathcal{A}$ . For an almost disjoint family  $\mathcal{A}$ , let  $\Psi(\mathcal{A}) := \mathcal{A} \cup \mathbb{N}$ . A topology on  $\Psi(\mathcal{A})$  is defined as follows. The natural numbers are isolated, and for each element  $A \in \mathcal{A}$  and each finite set  $F \subseteq \mathbb{N}$ , the set  $\{A\} \cup (A \setminus F)$  is a basic open neighborhood of  $A$ . Spaces constructed in this manner are called  *$\Psi$ -spaces*.

For a set  $X$ , a subset  $A$  of  $X$  and a family  $\mathcal{U}$  of subsets of  $X$ , let  $\text{star}(A, \mathcal{U}) := \bigcup \{U \in \mathcal{U} : A \cap U \neq \emptyset\}$ . A topological space  $X$  is *star-Lindelöf* if every open cover  $\mathcal{U}$  of  $X$  has a countable subset  $\mathcal{V}$  such that  $X = \text{star}(\bigcup \mathcal{V}, \mathcal{U})$ . It is easy to see that uncountable  $\Psi$ -spaces are not Lindelöf. Being separable, though, all  $\Psi$ -spaces are star-Lindelöf.

*Menger’s property* is the following selective version of Lindelöf’s property: For every sequence  $\mathcal{U}_1, \mathcal{U}_2, \dots$  of open covers of  $X$ , there are finite sets  $\mathcal{F}_1 \subseteq \mathcal{U}_1, \mathcal{F}_2 \subseteq \mathcal{U}_2, \dots$  such that the family  $\{\bigcup \mathcal{F}_1, \bigcup \mathcal{F}_2, \dots\}$  covers  $X$ .

A topological space  $X$  is *star-Menger* if for every sequence  $\mathcal{U}_1, \mathcal{U}_2, \dots$  of open covers of  $X$ , there are finite sets  $\mathcal{F}_1 \subseteq \mathcal{U}_1, \mathcal{F}_2 \subseteq \mathcal{U}_2, \dots$  such that the family  $\{\text{star}(\bigcup \mathcal{F}_1, \mathcal{U}_1), \text{star}(\bigcup \mathcal{F}_2, \mathcal{U}_2), \dots\}$  covers  $X$ .

## REFERENCES

- [1] M. Bonanzinga, M. Matveev, *Some covering properties for  $\Psi$ -spaces*, *Matematicki Vesnik* **61** (2009), 3–11.
- [2] B. Tsaban, *Combinatorial aspects of selective star covering properties in  $\Psi$ -spaces*, *Topology and its Applications* **192** (2015), 198–207.